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ABSTRACT

This teacher's guide for a semester course in analytic geometry is based on the text "Analytic Geometry" by W. K. Morrill. Included is a daily schedule of suggested topics and homework assignments. Specific teaching hints are also given. The content of the course includes point and plane vectors, straight lines, point and space vectors, planes, straight lines in space, circles, conics, transformation of axes, and polar coordinates. (Author/CT)

BALTIMORE COUNTY PUBLIC SCHOOLS

A Tentative Guide

ANALYTIC GEOMETRY

TOWSON, MARYLAND

JANUARY, 1967

BALTIMORE COUNTY PUBLIC SCHOOLS

A Tentative Guide

ANALYTIC GEOMETRY

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1967

ANALYTIC GEOMETRY

Analytic Geometry is a second semester course in the Trig-Analytics sequence. It is designed for those students who have successfully completed Algebra II, Geometry and Trigonometry. This course is also prerequisite for those students desiring the Calculus in the 12th year.

The text for the course is Analytic Geometry by W. K. Morrill which utilizes a vector approach to the subject. However, the use of vector techniques in no way implies that this should become a course in vector algebra. The page references are keyed to both the old and revised editions.

The guide is tentative and subject to continuous teacher evaluation. The time allotment, sequence of topics and homework assignments are suggestive. The teacher may adjust these according to the needs of the students. For instance, some teachers may wish to interchange the units on Transformation of Axes and Polar Coordinates. Teachers will want to avail themselves of appropriate overhead transparencies in the Geometry and Calculus series, which may be found in the department files. Standardized tests are also available at the Central Testing Office. These may be obtained through your department chairman.

The Board of Education and the Superintendent of Schools wish to express their appreciation to the Curriculum Committee and to all mathematics teachers of Baltimore County whose efforts made possible the development of this curriculum guide.

Special commendation is due Mrs. Betty Hagedorn, secretary to the committee, for her careful and painstaking efforts in the production of this guide.

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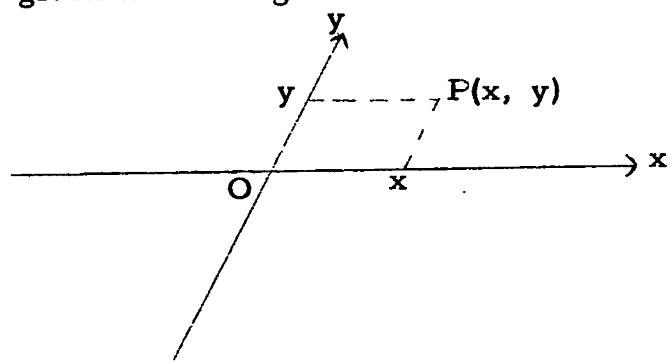
	UNIT ONE The Point and Plane Vectors		
1	Introduction (2-1) Emphasize the fundamental concept of analytic geometry: a point is a member of the graph of a relation if and only if its coordinates satisfy the equation of the relation.	MRev... M......	19 18
	The Rectangular Coordinate System Note that the choice of a <u>rectangular</u> system of coordinates is arbitrary. A different system is often more useful for the solution of specific problems. An example of an alternate system of axes is given in the diagram below.  THE OBLIQUE SYSTEM OF COORDINATES Emphasize the differences between a right-hand coordinate system and a left-hand coordinate system. Note that P(x, y) represents <u>any</u> point in the plane and not only those in quadrant I.	MRev... M......	19-21 18-20

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
1 (cont'd)	Symmetry (2-3) Symmetry with respect to a point Symmetry with respect to a line SUGGESTED BASIC ASSIGNMENT: The problems in the Suggested Basic Assignment are <u>not</u> meant to constitute the entire assignment. They are problems which should be included in assignment. MRev... Pp. 21, 22: Ex. 3, 6, 8, 10, 13, 18, 20, 23, 25, 26 M...... Pp. 20, 21: Ex. 3, 6, 8, 10, 13 MRev... P. 23: Ex. 2 M...... Pp. 21, 22: Ex. 2	MRev... M......	22-23 21
2	Projections Students who have studied the SMSG Geometry consider the projection of a point or segment in a line to be a set of points. The text defines the projection of a directed segment P_1P_2 in a line L to be the directed distance $\overline{AB}$ where A is the projection of P_1 in L and B is the projection of P_2 in L. Observe that in SMSG Geometry $\overline{AB}$ represents a segment with A and B as end points and AB represents the undirected length of $\overline{AB}$. However, note that in the text all segments are directed segments. P_1P_2 represents the directed segment from P_1 to P_2; $\overline{AB}$ represents the directed distance from A to B where A and B are points in the x or y axes, and $\vert P_1P_2\vert$ represents the length of P_1P_2. Indicate the differences between the notation used in SMSG and the text to avoid confusion.	MRev... M......	23-25 22-24

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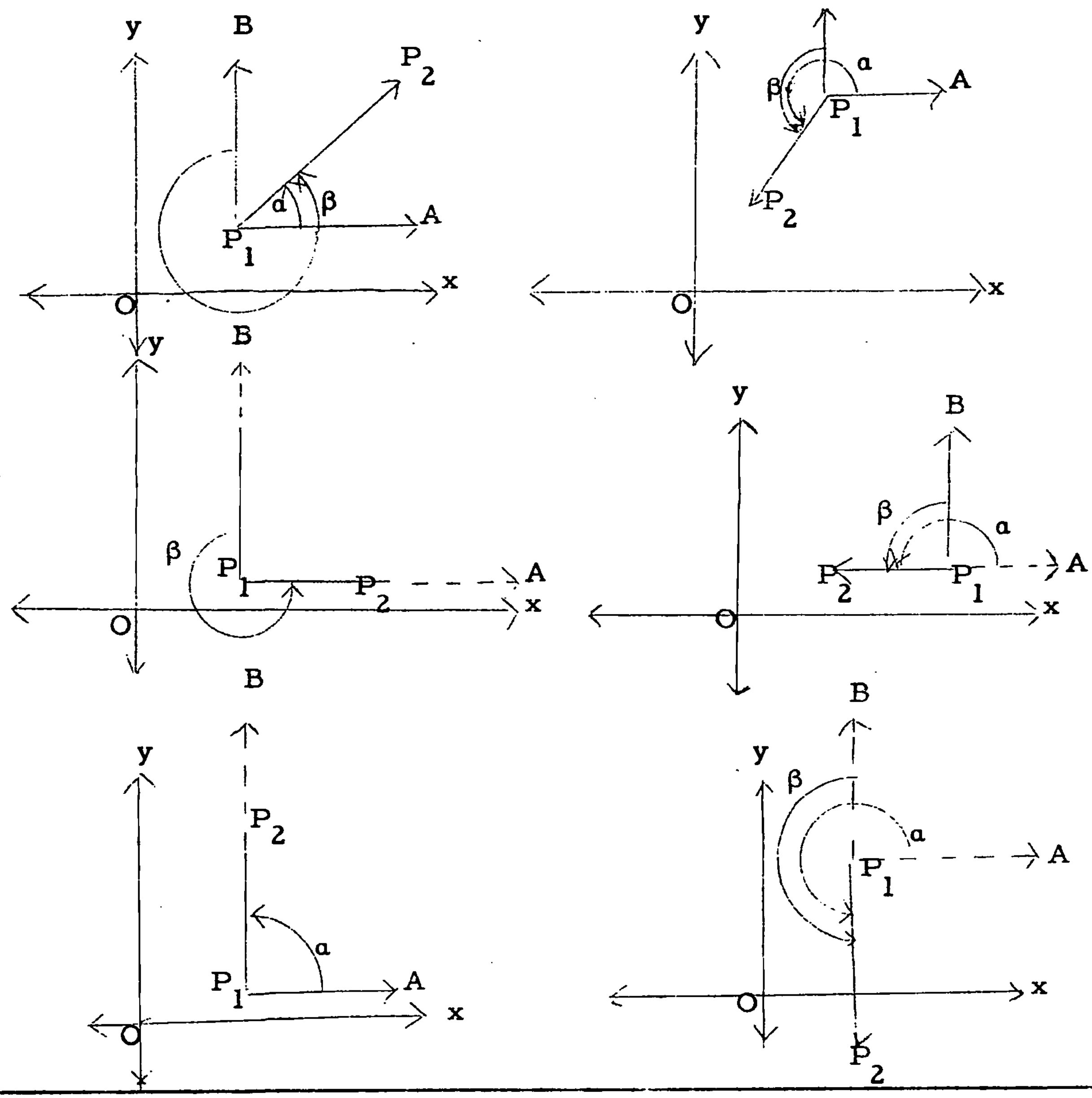
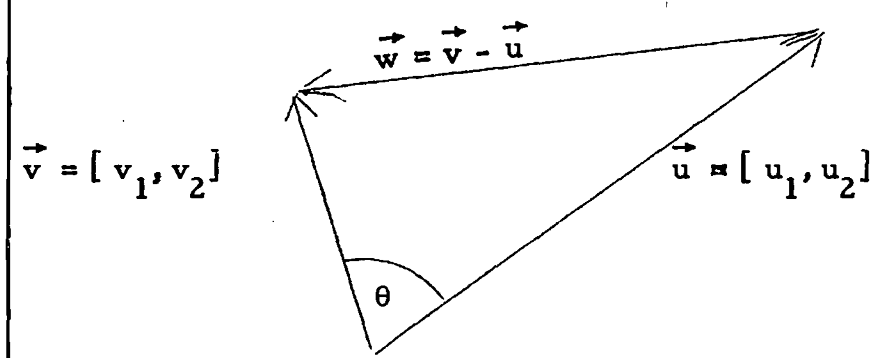
2 (cont'd)	Scalar Components of a Segment (2-5) SUGGESTED BASIC ASSIGNMENT: MRev... P. 25: Ex. 1, 3 MRev... P. 27: Ex. 1, 3, 4 M...... Pp. 23, 24: Ex. 1, 3 M...... Pp.25, 26: Ex. 1, 3, 4	MRev... M......	26-27 24-25
3	Distance Between Two Points (2-6) SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 29, 30: Ex. 2a, 3, 6, 7, 8, 9, 10, 12, 13 M...... P. 28: Ex. 2a, 3, 6, 7, 8	MRev... M......	27-30 26-28
4	Direction Cosines of a Segment (2-7) Note that the text does not define direction angles of a segment. This should be done as follows: The <u>direction angles of</u> $\overline{P_1P_2}$ where $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are the angles measured in a counterclockwise direction from two rays $\overrightarrow{P_1A}$ and $\overrightarrow{P_1B}$ parallel to the positive x and y axes respectively to the directed segment P_1P_2. These angles are denoted by α and β respectively. Illustrate these definitions with diagrams similar to the ones below.	MRev... M......	31-33 29-31

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
4 (cont'd)			

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
4 (cont'd)	The direction cosines, l and m, of P_1P_2 are defined to be the cosines of α and β respectively. Emphasize the advantages of the cosine function over the other trigonometric functions. SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 32, 33: Ex. 1, 2, 3, 4, 5, 8 M...... Pp. 30, 31: Ex. 1, 2, 3, 4, 5, 8		
5	Plane Vectors (2-8) Vectors are central in the development of the text. They are abstract concepts, being sets of equivalent directed segments. This is similar to the idea of equivalence classes. The analogy should be pursued with students familiar with the concept. Students of physics are familiar with the vector concept. However, vectors have been defined for them as "quantities possessing magnitude and direction". This is a geometrical interpretation and is what our text means by a representative of the vector. A <u>scalar</u> is a real number. Examples of scalars are length, area, volume, time, mass, and temperature. Examples of physical quantities that are represented by vectors are force, velocity, and acceleration. Indicate the similarity between the algebra of real numbers and the algebra of vectors. <u>However, do not prove all the algebraic properties of vectors.</u> The purpose of this course is to develop analytic geometry, not vector algebra. EXAMPLE: Express [3, 4] in terms of [1, 0] and [0, 1]. SOLUTION: [3, 4] = [3, 0] + [0, 4] = 3[1, 0] + 4[0, 1].	MRev... M......	33-36 31-34

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
5 (cont'd)	Several examples like this will lead to the generalization that the vectors [1, 0] and [0, 1] are the "building blocks" of the system of vectors. SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 36, 37: Ex. 1, 2a, 3, 4, 5, 6, 10, 11, 12 Omit number 8. M...... P. 34: Ex. 1, 2a, 3, 4, 5, 6, 10(and 11, 12 of MRev.)		
6	Angle Between Two Vectors (2-9) An alternate proof of the formula for determining the angle between two vectors is given below and may be used if the teacher prefers it to the one in the text. THEOREM: If $\vec{u} = [u_1, u_2]$ and $\vec{v} = [v_1, v_2]$ are two non-zero vectors, the angle θ between them is given by $\cos\theta = \dfrac{u_1 v_1 + u_2 v_2}{\lvert\vec{u}\rvert\,\lvert\vec{v}\rvert}$ PROOF: Let $\vec{w} = \vec{v} - \vec{u}$. Thus, $\vec{w} = [v_1 - u_1, v_2 - u_2]$ By the law of cosines (see diagram), $\lvert\vec{w}\rvert^2 = \lvert\vec{u}\rvert^2 + \lvert\vec{v}\rvert^2 - 2\lvert\vec{u}\rvert\,\lvert\vec{v}\rvert\cos\theta$	MRev... M......	37-39 34-36

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
6 (cont'd)	 But, since $\vec{w} = [v_1 - u_1, v_2 - u_2]$, $\lvert [v_1 - u_1, v_2 - u_2] \rvert^2 = \lvert\vec{u}\rvert^2 + \lvert\vec{v}\rvert^2 - 2\lvert\vec{u}\rvert\lvert\vec{v}\rvert \cos\theta$ $(v_1 - u_1)^2 + (v_2 - u_2)^2 = \lvert\vec{u}\rvert^2 + \lvert\vec{v}\rvert^2 - 2\lvert\vec{u}\rvert\lvert\vec{v}\rvert \cos\theta$ $(v_1^2 - 2v_1u_1 + u_1^2) + (v_2^2 - 2v_2u_2 + u_2^2) = \lvert\vec{u}\rvert^2 + \lvert\vec{v}\rvert^2 - 2\lvert\vec{u}\rvert\lvert\vec{v}\rvert \cos\theta$ $(v_1^2 + v_2^2) + (u_1^2 + u_2^2) - 2(u_1v_1 + u_2v_2) = \lvert\vec{u}\rvert^2 + \lvert\vec{v}\rvert^2 - 2\lvert\vec{u}\rvert\lvert\vec{v}\rvert \cos\theta$ $\lvert\vec{v}\rvert^2 + \lvert\vec{u}\rvert^2 - 2(u_1v_1 + u_2v_2) = \lvert\vec{u}\rvert^2 + \lvert\vec{v}\rvert^2 - 2\lvert\vec{u}\rvert\lvert\vec{v}\rvert \cos\theta$ $-2(u_1v_1 + u_2v_2) = -2\lvert\vec{u}\rvert\lvert\vec{v}\rvert \cos\theta$ $\dfrac{u_1v_1 + u_2v_2}{\lvert\vec{u}\rvert\lvert\vec{v}\rvert} = \cos\theta$ Q. E. D.		

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(cont'd)</td><td>An elegant feature of the above proof is that there is no reference to any coordinate system and thus the proof generalizes to 3-space quite easily.

The teacher should note that the text's definition of the dot product is different from the one in <u>PSSC Physics</u>. It is suggested that the teacher consult the teacher of physics to avoid confusion in the term dot product.

SUGGESTED BASIC ASSIGNMENT:

MRev... Pp. 41, 42: Ex. 8, 16, 17, 18
M...... Pp. 38, 39: Ex. 8 (and 16, 17, 18 of MRev.)</td><td></td><td></td></tr>
<tr><td>7</td><td>Parallel and Perpendicular Vectors (2-10)

The teacher should study the concept of proportionality in Chapter 7, <u>Similarity</u>, of <u>SMSG Geometry with Coordinates</u>. This will clear up the apparent possibility of division by zero in the discussion on parallelism of vectors on page 40 of the text. To avoid this difficulty, the teacher may want to adopt the results of problem 9 on page 41 of MRev as the definition of parallel vectors.

SUGGESTED BASIC ASSIGNMENT:

MRev... Pp. 41, 42: Ex. 1a, 2, 5, 6, 9, 10, 11, 12, 14a, 19
M...... Pp. 38, 39: Ex. 1a, 2, 5, 6, 7 (and 9, 10, 11, 12, 14a, 19 of MRev.)</td><td>MRev...
M......</td><td>39-42
36-38</td></tr>
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LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
8	The Coordinates of a Point That Divides a Segment in a Given Ratio (2-11) The teacher should emphasize the solution of each problem by using an appropriate vector equation rather than by substitution in the formulas on page 44 of the text. In effect, each problem should be a redevelopment of the formulas on page 44. SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 46, 47: Ex. 1, 3, 15, 16, 17, 18, 19 M...... Pp. 43, 44: Ex. 1, 3, 15, 16, 17, 18, 19	MRev... M......	42-45 39-42
9	The Midpoint of a Segment and Analytic Proofs of Geometric Theorems (2-12) The teacher should spend most of the lesson solving problems such as numbers 9, 11, 13, and 14 on pages 46 and 47 of the text. SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 46, 47: Ex. 5, 7, 9, 10 M...... Pp. 43, 44: Ex. 5, 7, 9, 10	MRev... M......	45-47 42-44
10	Complementary Vectors (2-13)	MRev... M......	48 44-45
	Area of a Triangle and the Bar Product of Two Vectors (2-14) SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 50, 51: Ex. 1, 2, 3, 4b, 5a, 6, 7 Omit number 9 for students not familiar with determinants. M...... Pp. 47, 48: Ex. 1, 2, 3, 4b, 5a, 6, 7	MRev... M......	49-51 45-47

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
11	Review for Test and Review Exercises Be sure to include problems from the Review Exercises at the end of each chapter of the text. Besides providing good problems for review, material is often introduced or extended beyond the point covered earlier in the chapter.	MRev... M......	52-54 48-49
12	Test on Chapter 2		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
	UNIT TWO	
	The Straight Line	
13	Direction Numbers and Direction Cosines of a Line (3-1)	MRev... 55-56 M...... 50-51
	Emphasize that a line has an infinite number of pairs of direction numbers associated with it, while it has only two pairs of associated direction cosines.	
	EXAMPLE: The line $\overleftrightarrow{AB}$, with A(1, 2) B(-5, 6) has direction numbers: [-6, 4], [-3, 2], [3, -2], [9, -6] etc.	
	The direction cosines of $\overleftrightarrow{AB}$ are:	
	$\left[\frac{-6}{2\sqrt{13}}, \frac{4}{2\sqrt{13}}\right] = \left[\frac{-3}{\sqrt{13}}, \frac{2}{\sqrt{13}}\right]$	
	$\left[\frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}}\right] = \left[\frac{9}{3\sqrt{13}}, \frac{-6}{3\sqrt{13}}\right]$	
	The Slope of a Line (3-2)	MRev... 57-58 M...... 51-54
	Although slope is not a new concept to the student, inclination is. Inclination in the text is used in two contexts, the angle between the positively directed x axis and the line, measured in a counter-clockwise direction and as the measure of that angle.	
	SUGGESTED BASIC ASSIGNMENT:	
	MRev...P. 58: Ex. 1ab, 2, 3, 4, 5, 6ab M...... P. 54: Ex. 1ab, 2, 3, 4, 5, 6ab	

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
14	Parametric Equations of Lines (3-3) This concept is not new to those students who have studied SMSG Geometry. The vector approach of this text, however, should make for a better understanding. For students who did not have SMSG Geometry parametric equations will be a new concept. Students will need this method of writing linear equations in three dimensions so they should become adept in using this method now. Have students graph some lines whose equations are given in parametric form. (Exercise 2 at end of section.)	MRev... M......	59-61 54-56
15	Equations of Lines in Direction Number Form (Symmetric Form) (3-4) $x = x_1 + t\Delta x$, $y = y_1 + t\Delta y$ Parametric Form $\frac{x - x_1}{\Delta x} = \frac{y - y_1}{\Delta y}$ Symmetric Form As can be seen above the symmetric form of an equation of a line is easily derived from the parametric form by solving for the parameter. Symmetric form is not usable if either Δx or Δy is 0. Give students practice in converting from parametric form to symmetric form. Have the students do some of the exercises at the end of this section in symmetric form and leave their answers in that form. Rule 3-13 should be omitted. SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 61, 62: Ex. 1 (in parametric and symmetric form), 2, 6ab, 9ab M...... P. 56: Ex. 1 (in parametric and symmetric form), 2, and include 6ab, 9ab, of MRev.	MRev... M......	59-64 54-56

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
16	The General Linear Equation $ax + by + c = 0$ (3-5) Have the students thoroughly understand the two part proof of: a) The set of points $P(x, y)$ having the property that P_1P $[x - x_1, y - y_1]$ is perpendicular to $\vec{u}\,[a, b]$ containing P_1 has an equation $ax + by = c$. b) $ax + by = c$ is an equation of a line which has a vector $[a, b]$ perpendicular to it. This section is an exciting and powerful result of our previous vector treatment. SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 64, 65: Ex. 2ab, 3ab, 6ab M...... P. 59: Ex. 2ab, 3ab, (and 6ab of MRev.)	MRev... M......	65-68 59-63
17	Lines Perpendicular to and Parallel to a Given Line (3-11) This section and the exercises which follow give further emphasis to Section (3-5). SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 81, 82: Ex. 1abcd, 2ab, 3ab, 4ab M...... Pp. 73, 74: Ex. 1abcd, 2ab, 3ab, 4ab	MRev... M......	80-82 72-73

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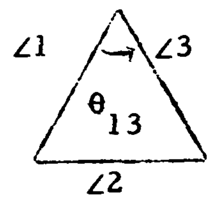
Angle Between Two Lines (3-6)

The teacher should give equal emphasis to finding the angle between two lines through the cosine formula and through the tangent formula. The cosine method is already familiar to the students from Chapter 2 and may be preferred by them. However, the tangent method should be encouraged as it reinforces the inclination concept and is useful in the calculus. The teacher should encourage drawings from students and give them exercises where a specified angle's cosine or tangent is required. (Problems 8, 10, 11, 12 at end of this section.) A good memory technique for finding the tangent of an angle is:



$$\text{Tan } \theta_{13} = \frac{s_3 - s_1}{1 + s_1 s_3}$$

Note that the symbol "θ_{13}" means the directed angle between $\angle_1$ and $\angle_3$.

SUGGESTED BASIC ASSIGNMENT:

MRev...Pp. 72, 73: Ex. 1a, 3a, 5, 6a, 8, 10, 11, 12
M...... P. 65: Ex. 1a, 3a, 5, 6a, 8 (and 10, 11, 12 of MRev.)

</td><td>MRev...
M......</td><td>70-71
63-65</td></tr>
<tr><td>19</td><td>Point Slope Form of Equation of Line (3-7)</td><td>MRev...
M......</td><td>73-75
65-68</td></tr>
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19 (cont'd)	Intercept Form of Equation of Line (3-8) Slope, Intercept Form of Equation of a Line (3-9) The three forms above should be a review for the students. SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 78, 79: Ex. 1abc, 2, 4, 13, 18, 19 M...... Pp. 71, 72: Ex. 1abc, 2, 4 (and 13, 18, 19 of MRev.)		
20	Distance from Line to Point (3-12) The teacher should emphasize: a. If the number $\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} > 0$, then the point $P(x_1, y_1)$ lies on the side of the line $ax + by + c = 0$ toward which the coefficient vector $[a, b]$ points, provided $a > 0$. b. If the number $\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} < 0$, then the point $P(x_1, y_1)$ lies on the side of the line opposite to that which the coefficient vector points. SUGGESTED BASIC ASSIGNMENT: MRev... P. 85: Ex. 3a, 4a, 5a, 8, 9 M...... Pp. 76, 77: Ex. 3a, 4a, 5a (and 8, 9 of MRev.)	MRev... M......	83-85 74-76

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
21	**Intersecting Lines (3-13)** This section gives an opportunity to introduce the class to determinants; evaluation of a second order determinant and its use for finding the point of intersection of 2 lines. If the class finds this easy or a review, it is advisable to evaluate third order determinants as they will be useful in the chapters which follow. The diagonal method of expansion may be used in 2nd order and 3rd order determinants. Expansion by cofactors is necessary for the expansion of all determinants beyond 3rd order. Encourage students to use determinants in the solution of problems 3, 4, 5 at the end of this section. SUGGESTED BASIC ASSIGNMENT: MRev... P. 90: Ex. 3, 4, 5 M..... P. 80: Ex. 3, 4, 5	MRev... M.....	86-88 78-80
22	**Bisectors of Angles (3-15)** The set of points equidistant from the sides of an angle is a ray which bisects the angle. The teacher should have the students adopt this "locus" theorem to their work in Section 3-15. Special care should be given to making a drawing and understanding which sign should be used in the problem at hand from the general equation for the angle bisector $\dfrac{a_1x + b_1y + c}{\sqrt{a_1^2 + b_1^2}} = \pm \dfrac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$	MRev... M.....	91-93 81-83

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
22 (cont'd)	Note: Illustrative example 3-26 should read: Find an equation of the bisector of the angle such that: 1) the angle is formed by the lines $l_1 = 3x - 4y + 10 = 0$ and $l_2 = 4x - 3y + 12 = 0$ 2) the origin is an interior point of the angle. SUGGESTED BASIC ASSIGNMENT: MRev. . . P. 93: Ex. 1, 3, 6, 8, 10 M. . . . . P. 83: Ex. 1, 3 (and 6, 8, 10 of MRev.)		
23	Review Since Sections 3-10, 3-14, 3-16, 3-17 have been omitted, review exercises 10, 19, 20, 30, 35, 36 should be omitted.		
24	Test		

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25	The Point in Space (9-1)	MRev... M......	297-298 277-278
	Projections (9-2)	MRev... M......	298-299 278-279
	Note the dual usage of "projection of a segment" as meaning both a segment and a directed distance.		
	SUGGESTED BASIC ASSIGNMENT: MRev... P. 300: Ex. 2, 5ae, 7, 9, 10 M...... P. 280: Ex. 2, 5ae, 7, 9, 10		
26	Scalar Components of a Segment (9-3)	MRev... M......	301 281
	Length, or Magnitude, of a Segment (9-4)	MRev... M......	301-303 281-283
	Direction Cosines of a Segment (9-5)	MRev... M......	303-304 283-284
	SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 304,305: Ex. 1a, 5a, 6, 7, 10 M...... Pp. 284,285: Ex. 1a, 5a, 6, 7, 10		
27	Space Vectors (9-6)	MRev... M......	305-309 285-288

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
27 (cont'd)	Cosine of the Angle Between Two Vectors (9-7) SUGGESTED BASIC ASSIGNMENT: MRev... P. 308: Ex. 3, 4ac, 6a, 7, 10a M...... P. 288: Ex. 3, 4ac, 6a, 7, 10a MRev... P. 312: Ex. 1, 2a, 3a, 9 M...... Pp. 291-292: Ex. 1, 2a, 3a, 9	MRev... M......	309-312 288-291
28	The Coordinates of a Point that Divides a Segment in a Given Ratio (9-8) The Midpoint of a Segment (9-9) SUGGESTED BASIC ASSIGNMENT: MRev... P. 315: Ex. 1, 2, 5, 7 M...... Pp. 294-295: Ex. 1, 2, 5, 7	MRev... M...... MRev... M......	312-314 292-294 314-315 294
29	Review		
30	Test		

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	UNIT FOUR The Plane		
31	An Equation of a Plane (10-1)	MRev...	318-319
		M......	297-298

It should be stressed that the equation for a plane in space is a linear equation in one or more of the variables x, y, z. The general equation of a plane is $Ax + By + Cx + d = 0$. The teacher should derive carefully the concept that for this equation the normal vector $[A, B, C]$ is $\perp$ to the plane.

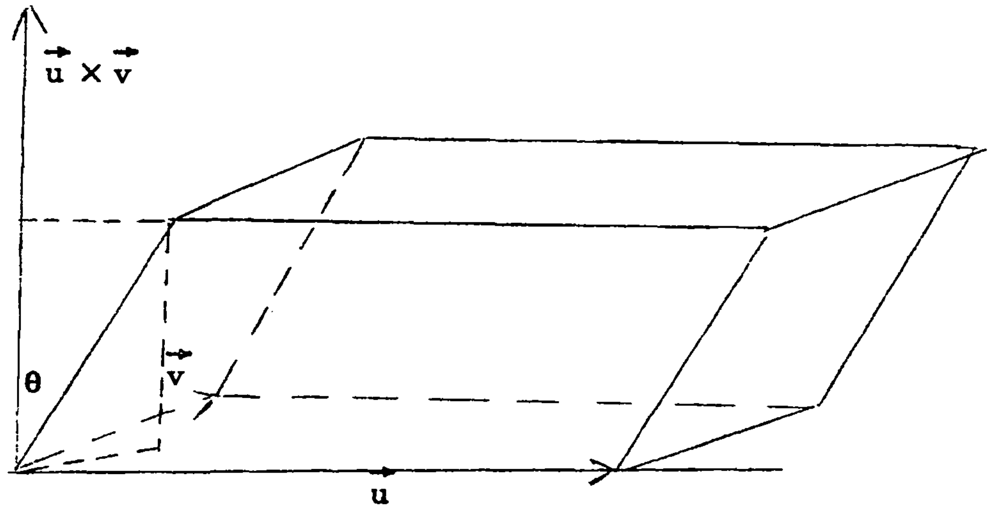
The teacher should emphasize that:

(1) in a one dimensional system
$x = 5$
is the equation of a point.

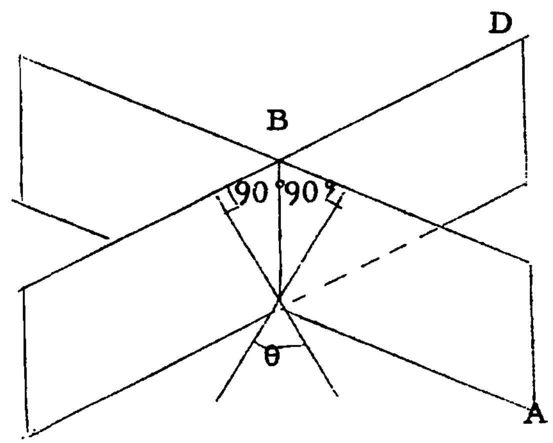
(2) in a two dimensional system
$x = 6$
$y = 3$
$x + 2y = 12$
are equations of lines.

(3) in a three dimensional system
$x + y = 6$
$x + z = 3$
$y + z = 9$
$x + 3y + 4z = 6$
are equations of planes.

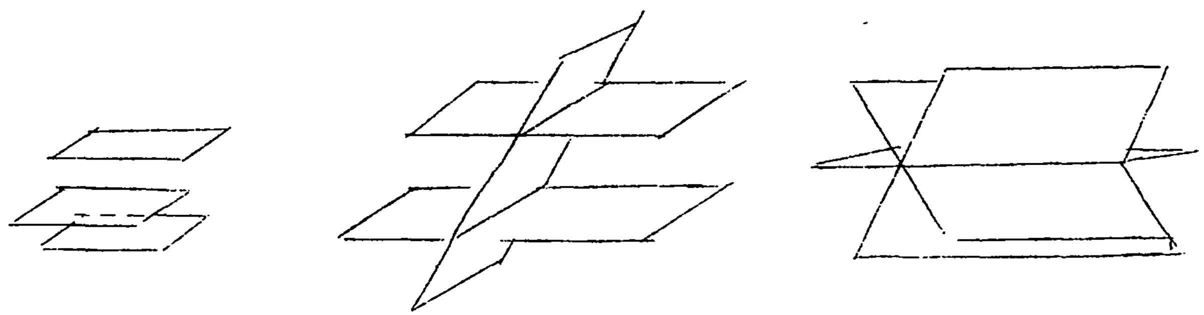
LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
	SUGGESTED BASIC ASSIGNMENT: MRev... P. 320: Ex. 2, 4, 6, 9, 10, 11, 12 M...... P. 299: Ex. 2, 4, 6, 9, 10, 11, 12		
32	Parallel and Perpendicular Planes (10-2) SUGGESTED BASIC ASSIGNMENT: MRev... P. 322: Ex. 1, 2, 3 M...... P. 301: Ex. 1, 2, 3	MRev... M......	321 300
33	Intercept Equation of a Plane (10-3) The teacher should give practice in the graphing of an equation by the intercept method. SUGGESTED BASIC ASSIGNMENT: MRev... P. 323: Ex. 1, 2 M...... P. 302: Ex. 1, 2	MRev... M......	322-323 301-302
34 - 5	Vector Product of Two Vectors (10-4) The teacher should stress: (1) the vector product is a vector and not a number (2) $\vec{u} \times \vec{v}$ is perpendicular to both $\vec{u}$ and $\vec{v}$ (3) $\vec{u} \times \vec{v}$ is the normal of the plane containing $\vec{u}$ and $\vec{v}$. The teacher should develop carefully the theorem on page 324 (304) and be sure to include the area of parallelograms.	MRev... M......	323-326 302

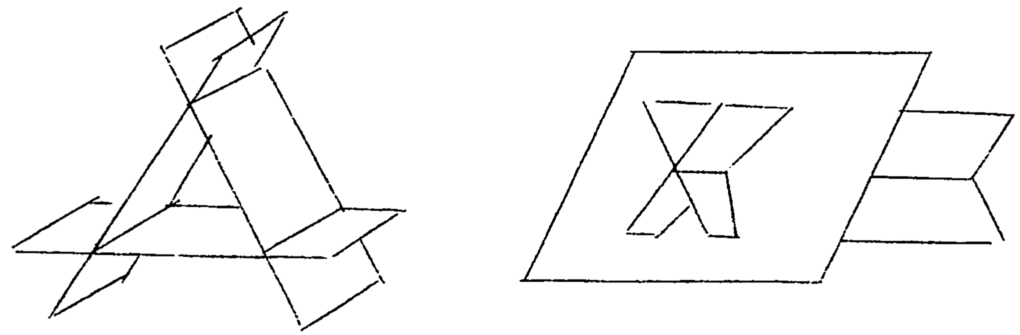
LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
36	Alternating Product or Scalar Triple Product of Three Vectors (10-5) The teacher should prove in class Example 8, page 327. (M. . . p. 305)  Additional properties of the scalar triple product are: (1) $\vec{u}$, $\vec{v}$, and $\vec{w}$ are coplanar if and only if $\vec{u} \cdot (\vec{v} \times \vec{w}) = 0$ (2) the value of the scalar triple product is not changed by a cyclic permutation of the elements which form the product. $\vec{u} \cdot (\vec{v} \times \vec{u}) = \vec{w} \cdot (\vec{u} \times \vec{v}) = \vec{v} \cdot (\vec{w} \times \vec{u})$ (3) the dot and cross in a scalar triple product can be interchanged without affecting the value of the product. $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w}$ SUGGESTED BASIC ASSIGNMENT: MRev. . . Pp. 326, 327: Ex. 2, 4, 5, 6, 7 M. . . . . P. 305: Ex. 2, 4, 5, 6, 7	MRev. . . M.	326 305

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
37	Determining a Plane Satisfying Three Conditions (10-6) The illustrations in the text are based on the theorem at the bottom of the page 328. An alternate method is to find the cross product of two of the vectors which results in the normal of the plane of these two vectors. Follow Example 10-1 on page 319. SUGGESTED BASIC ASSIGNMENT: MRev... P. 330: Ex. 1, 2, 3, 4 M...... Pp. 307, 8: Ex. 1, 2, 3, 4	MRev... M......	328 305
38	Distance From a Plane to a Point (10-7) SUGGESTED BASIC ASSIGNMENT: MRev... P. 333: Ex. 1, 2, 3 M...... P. 310: Ex. 1, 2, 3	MRev... M......	331 308
39	Angle Between Two Planes (10-8) The magnitude of the four dihedral angles formed by two intersecting planes are numerically equal, respectively, to the four corresponding angles formed by the two lines that can be drawn through any point in space perpendicular to the given planes.	MRev... M......	334 310

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
39 (cont'd)	 The teacher should stress that the angle formed by two planes is found by computing the angle between the two normals and is similar to the theorem in Geometry which states "two angles whose sides are respectively perpendicular are equal." SUGGESTED BASIC ASSIGNMENT: MRev... P. 336: Ex. 1 M...... P. 312: Ex. 1		
40	Review		
41	Test		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
	UNIT FIVE Straight Line in Space		
42	Direction Numbers and Direction Cosines (11-1) Since the analogy to Sections 9-3 and 9-5 is so very marked, only a brief discussion of this section should be necessary. Nevertheless a discussion should ensue for review and reinforcement purposes.	MRev... M......	338 315
	Parametric Equations of a Line (11-2) SUGGESTED BASIC ASSIGNMENT: MRev... P. 339: Ex. 1a, 1b, 2a, 3, 4 M...... P. 316: Ex. 1a, 1b, 2a, 2b, 3, 4 MRev... P. 342: Ex. 1a, 1c, 2a M...... P. 320: Ex. 1a, 1c, 2a	MRev... M......	340 317
43	Symmetric Equations of a Line (11-3)	MRev... M......	341 318
	The General Equations of a Line (11-4) The most common error made by students is that of thinking that since a first degree equation in two variables represents a line in two dimensions, then a first degree equation in three variables represents a straight line in space. Great emphasis should be placed on the fact that this is not true. Emphasize that a line in space cannot be described by less than two first degree equations. Indeed, it should be apparent that even when two equations are	MRev... M......	341 318

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
43 (cont'd)	used, this does not represent a unique description. Since an infinite number of planes pass through a given line, any two such planes considered together can be used to represent the line. Show that the cross product of the coefficient vectors of two planes whose intersection describes the line determines a triple of direction numbers for that line. Students should be facile in changing from the general to the symmetric and parametric forms. SUGGESTED BASIC ASSIGNMENT: MRev... P. 343: Ex. 3ad, 4a, 5, 6, 7, 8, 9, 15b M...... P. 340: Ex. 3ad, 4a, 5, 6, 7, 8, 9, 15b		
44	Intersecting Planes (11-5) The development and practice in spatial perception is particularly evident in this section. Students should be asked to submit sketches showing the different possibilities in which three planes in space can intersect. 	MRev... M......	345 322

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
44 (cont'd)	 SUGGESTED BASIC ASSIGNMENT: MRev... P. 347: Ex. 1, 2, 3, 4 M...... P. 324: Ex. 1, 2, 3, 4		
45	Review SUGGESTED BASIC ASSIGNMENT: MRev... Review Exercises P. 352: Ex. 1, 2, 3, 4, 5, 9ab M...... Review Exercises P. 328: Ex. 1, 2, 3, 4, 5, 9ab These problems make use of the algebraic representations of both lines and planes. Great caution must be exercised by the student in keeping the two distinct.		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
45 (cont'd)	The results in the answer section do not always make use of the obvious, given point in writing the equations for a line. Nevertheless the results are correct and this apparent discrepancy illustrates again the wide choice of expressions which one has available to describe a line in space. Students may wish to work for the answer given by the author, but it should be understood that this is not obligatory.		
46	Test		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
	UNIT SIX **The Circle**		
	Introduction By some definitions a circle is a conic section; by others, it is not. Refer to Sections 5-1 and 5-2. The word "radius" is used in this text both as a line segment and as a distance.		
47	Standard Equation (4-1)	MRev... M......	103-104 91-93
	The General Equation of a Circle (4-2) In previous Geometry courses the students have had some work with writing equations of circles where the center and radius is given. They will also be familiar with finding the center and radius of a circle when the equation of a circle is given. A drill would help the teacher discover the students' background on this subject. SUGGESTED BASIC ASSIGNMENT: MRev... P. 105: Ex. 1f, 2c, 4c, 5ac, 6c, 8 M...... P. 93: Ex. 1f, 2c, 4c, 5ac, 6c (and 8 of MRev.) MRev... P. 108: Ex. 1, 11, 13, 16 M...... Pp. 95, 96: Ex. 1, 11, 13 (and 16 of MRev.)	MRev... M......	106-107 93-95

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
48	Circle Determined by Three Conditions (4-3) An alternate method to the determinant solution given in the text is suggested as follows: (1) To find the equation of a circle which passes through three given points, find the equations of the perpendicular bisectors of two segments determined by the points. (2) The points of intersection of the perpendicular bisectors is the center of the circle. (3) The radius of the circle will be the distance from the center to one of the given points. SUGGESTED BASIC ASSIGNMENT: MRev...P. 111: Ex. 1ah, 4ah M...... Pp. 98, 99: Ex. 1ah, 4ah	MRev... 108-111 M...... 96-99
49	Symmetry (4-4)	MRev... 112-114 M...... 99-100
	Tangents to a Circle from an External Point (4-5)	MRev... 114-115 M...... 100-101
	Length of a Tangent from a Point Outside a Circle to the Circle (4-6) SUGGESTED BASIC ASSIGNMENT: MRev...P. 117: Ex. 1, 2, 3, 4, 5 M...... Pp. 104, 105: Ex. 1, 2, 3, 4, 5	MRev... 115-116 M...... 102-103

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
50	Radical Axis (4-7)	MRev...	117-121
		M......	104-108
	SUGGESTED BASIC ASSIGNMENT:		
	MRev... P. 121: Ex. 1ac, 2ab, 3ac		
	M...... P. 108: Ex. 1ac, 2ab, 3ac		
51	Equation of the Tangent to the Circle $x^2 + y^2 = r^2$ at a Point on the Circle (4-8)	MRev...	122-123
		M......	109-110
	Equation of the Tangent to the Circle $x^2 + y^2 = r^2$ when the Slope of the Tangent is Known (4-9)	MRev...	124-126
		M......	111-113
	The student should not memorize any special formulas to do the problems in this section. Instead, each problem should be solved using vector methods.		
	Parametric Equations of the Circle (4-11)	MRev...	132-133
		M......	118-120
	SUGGESTED BASIC ASSIGNMENT:		
	MRev... Pp. 123, 124: Ex. 1af, 3af, 5af		
	M...... P. 110: Ex. 1af, 3af, 5af		
	MRev... P. 126: Ex. 1a, 2a, 3a		
	M...... P. 113: Ex. 1a, 2a, 3a		
52	Review		
	SUGGESTED BASIC ASSIGNMENT:		
	MRev... Pp. 133, 135: Ex. 1 (a-h inclusive), 3, 5, 11, 16		
	M...... Pp. 120, 121: Ex. 1 (a-h inclusive), 3, 5, 11, 16		
53	Test		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE | PAGE
	UNIT SEVEN The Conics	
54	The Conic as a Section of a Cone (5-1) General Definition of a Conic (5-2) The teacher should be aware that when $e = 0$ the definition fails. But when e approaches zero, the ellipse approaches a circle as a limit. Hence, a circle is a special kind of ellipse. SUGGESTED BASIC ASSIGNMENT: MRev... P. 141: Ex. 1, 2, 3, 4, 9, 10, 12 M...... P. 127: Ex. 1, 2, 3, 4, 9, 10, 12	MRev... 138-139 M...... 124-127
55	The Parabola Explanation of Terms (5-3) A Geometric Construction of the Parabola (5-4) Simple Equations of the Parabola (5-5) SUGGESTED BASIC ASSIGNMENT: MRev... P. 146: Ex. 4, 5, 6, 7a, 8a M...... P. 132: Ex. 4, 5, 6, 7a, 8a	MRev... 142-145 M...... 128-131
56	The Latus Rectum (5-6)	MRev... 146-147 M...... 132-133

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
56 (cont'd)	SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 147, 148: Ex. 3, 4 M...... Pp. 133, 134: Ex. 3, 4		
57	Parabolic Arch (5-7) Parametric Equations of the Parabola (5-8) Applications of Parabolic Curves (5-9) SUGGESTED BASIC ASSIGNMENT: MRev... P. 151: Ex. 1, 2, 3 M...... P. 137: Ex. 1, 2, 3	MRev... M......	148-151 134-137
58	Simple Equations of the Ellipse (5-10) Compare the definition of the ellipse with that of the parabola, noting the value of "e".	MRev... M......	153-154 139-140
59	Simple Equations of the Ellipse (continued) Develop the simple equation of the ellipse. (The set of calculus projectuals contains a transparency on this topic. However, it is not based on the definition of the ellipse which is used in our text.) Apply $b^2 = a^2 - c^2$ and $c = ae$ to the writing of simple equations of the ellipse.	MRev... M......	154-156 140-143

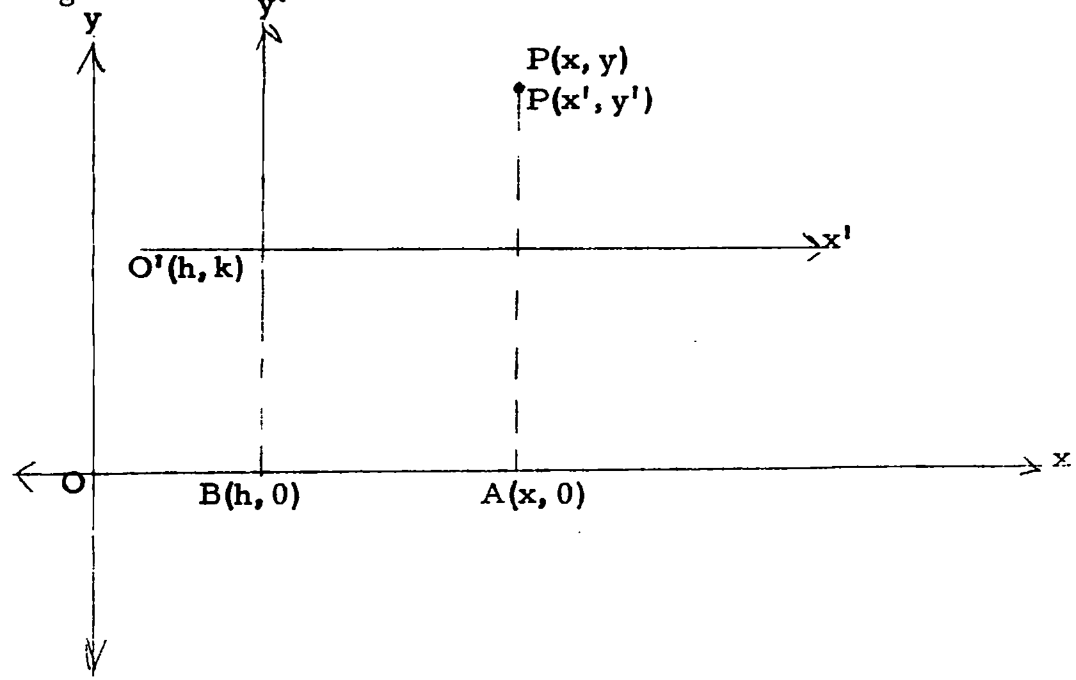
LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
59 (cont'd)	SUGGESTED BASIC ASSIGNMENT: MRev... P. 157: Ex. 1, 4, 5ace M...... Pp. 142, 143: Ex. 1, 4, 5ace		
60	Explanation of Terms for the Ellipse (5-11)	MRev... M......	157-158 143-144
	The Focal Radii (5-12) SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 158, 159: Ex. 1, 3, 5, 7, 9 M...... P. 144: Ex. 1, 3, 5, 7, 9 MRev... P. 159: Ex. 2 M...... P. 145: Ex. 2	MRev... M......	159 145
61	Discussion of the Equation $Ax^2 + Cy^2 + F = 0$ where A and C are of Like Sign and are Not Equal to Zero (5-13) Clarify the meaning of "discussion" of an equation. (Refer to Ex. 5-5, MRev... P. 161, M...... P. 147.) Define point ellipse and imaginary ellipse. Special note should also be made at this time concerning the circle as a special ellipse. (Refer to MRev... P. 161, M...... P. 146.)	MRev... M......	160-162 145-147
	The Latus Rectum of an Ellipse (5-14) SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 163, 164: Ex. 1ab, 2ace, 3ace, 4a, 5 M...... P. 149: Ex. 1ab, 2ace, 3ace, 4a, 5	MRev... M......	162-163 148

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
62	Construction of an Ellipse (5-15)	MRev...	165-167
		M......	150-151
	The Construction in Method 1 is an Application of 5-12		
	Parametric Equations of the Ellipse (5-16)	MRev...	167
	These follow directly from construction method 2.	M......	152
	Applications of the Ellipse (5-17)	MRev...	168
	SUGGESTED BASIC ASSIGNMENT:	M......	152
	MRev... Pp. 163, 164: Ex. 1d, 2f, 3h, 4c		
	M...... P. 149: Ex. 1d, 2f, 3h, 4c		
	MRev... P. 167: Ex. 1ac		
	M...... P. 152: Ex. 1ac		
63	Simple Equation of the Hyperbola (5-18)	MRev...	168-170
	Use the definition of 5-2 to derive the simple equation of the hyperbola.	M......	152-154
	Explanation of Terms for the Hyperbola (5-19)	MRev...	171-173
	Students should become adept at the discussion of the simple hyperbola.	M......	155-157
	SUGGESTED BASIC ASSIGNMENT:		
	MRev... P. 170: Ex. 4		
	M...... P. 154: Ex. 4		
	MRev... P. 175: Ex. 1be, 2acei		
	M...... P. 159: Ex. 1be, 2acei		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
64	Asymptotes of Hyperbola (5-20) The definition of the asymptote as given in the text should be stressed in preference to such informal and incorrect phrases as "asymptote never touches the curve". This may be true for the asymptotes of a hyperbola; however, it is not true for asymptotes of such curves as $y = e^x \sin x$. SUGGESTED BASIC ASSIGNMENT: MRev... P. 175: Ex. 1df, 2flm, 3 M...... P. 159: Ex. 1df, 2flm, 3	MRev... M......	173-175 157-159
65	The Focal Radii (5-21) SUGGESTED BASIC ASSIGNMENT: MRev... P. 178: Ex. 1, 2 M...... P. 161: Ex. 1, 2	MRev... M......	176-177 160-161
66	Discussion of the Equation $Ax^2 + Cy^2 + F = 0$ where $A \neq C \neq 0$, and A and C are Opposite in Sign (5-22)	MRev... M......	178-179 161-163
	The Latus Rectum of the Hyperbola SUGGESTED BASIC ASSIGNMENT: MRev... P. 180: Ex. 2, 3 M...... P. 164: Ex. 2, 3	MRev... M......	179-180 163-164
67	Conjugate Hyperbolas (5-24)	MRev... M......	181 164
	Equilateral Hyperbolas (5-25)	MRev... M......	181 164-165

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
67 (cont'd)	SUGGESTED BASIC ASSIGNMENT: MRev... P. 181: Ex. 1abd, 2abceoqtx M...... P. 165: Ex. 1abd, 2abceoqtx		
68	Construction of the Hyperbola (5-26)	MRev... M......	183-184 166-167
	Applications of the Hyperbola (5-27)	MRev... M......	184 167
	Parametric Equations of a Hyperbola (5-28) SUGGESTED BASIC ASSIGNMENT: MRev... P. 185: Ex. 1ad, 2ad M...... P. 168: Ex. 1ad, 2ad	MRev... M......	184 167
69	Summary and Review (5-29) All the problems in the review exercises are good examples of the concepts of this unit. The assignment below represents only a minimum. SUGGESTED BASIC ASSIGNMENT: MRev... P. 186: Ex. 1abd, 2abceg, 4, 9 M...... P. 169: Ex. 1abd, 2abceg, 4, 9	MRev... M......	185-188 168-169
70	Test		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
	UNIT EIGHT Transformation of the Axes		
71	Introduction (6-1)	MRev... M......	192 174
	Translation of the Axes (6-2) The text uses a vector proof to obtain the formulas for the translation of the axes. However, the traditional scalar proof is used when the formulas for the rotation of the axes are derived. For consistency, the teacher may wish to use the following traditional derivation of the translation formulas.	MRev... M......	192-195 174-176



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<tr><th rowspan="2">LESSON</th><th rowspan="2">SCOPE AND TEACHING SUGGESTIONS</th><th colspan="2">REFERENCE</th></tr>
<tr><th>CODE</th><th>PAGE</th></tr>
<tr><td>71
(cont'd)</td><td>THEOREM: Let x-y be a rectangular coordinate system and let x'-y' be a second rectangular coordinate system which is a translation of the first system. If the coordinates of a point P relative to the x-y system are P(x, y) and the coordinates of P relative to the x'-y' system are $P(x', y')$, then

$x = x' + h$
and $y = y' + k$

where $O'(h, k)$ are the coordinates of the center of the x'-y' system with respect to the x-y system.

PROOF: Project P(x, y) onto the x-axis. Let A be the image of P in the x-axis where A has coordinates (x, 0). Similarly, the image of O' in the x-axis is B with coordinates (h, 0), by the definition of a translation. Thus,

$OA = OB + BA$
$x = h + x'$

or $x' = x - h.$

Similarly, $y' = y - k.$

Q. E. D.

SUGGESTED BASIC ASSIGNMENT:

MRev... Pp. 194, 195: Ex. 1ace, 2aeg, 3acde
M..... Pp. 176, 177: Ex. 1ace, 2aeg, 3acde</td><td></td><td></td></tr>
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<tr><td>72</td><td>Simplification of the Equation $Ax^2 + Cy^2 + Dx + Ey + F = 0$, Where Both A and C are Not Zero (6-3)

The formulas developed in this section should <u>not</u> be memorized. The student should learn the general procedure and apply it in each example. For instance, to eliminate the linear terms in
$4x^2 - y^2 + 24x + 2y + 36 = 0$

The student should replace x by $x' + h$ and y by $y' + k$ and determine the values of h and k which transforms the equation to one of the form
$Ax'^2 + Cy'^2 + F = 0.$

SUGGESTED BASIC ASSIGNMENT:

MRev... P. 199: Ex. 2, 4, 8, 15
M...... P. 181: Ex. 2, 4, 8, 15</td><td>MRev...
M......</td><td>195-200
177-182</td></tr>
<tr><td>73</td><td>The Standard Equations of the Conics (6-4)

The teacher should develop in class:
(1) the standard equations for the parabola
(2) either the standard equation of the ellipse or the standard equation of the hyperbola.
Emphasize that the point C with coordinates (h, k) is the <u>center</u> of the ellipse or hyperbola, but that point C with coordinates (h, k) is the <u>vertex</u> of the parabola in standard form. The students should be able to complete charts similar to the following for each conic.</td><td>MRev...
M......</td><td>200-205
182-187</td></tr>
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LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
73 (cont'd)	A DISCUSSION OF THE HYPERBOLA IN STANDARD FORM		

Equation	Center	Transverse Axis	Eccentricity	Vertices
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$c(h, k)$	$x = h$	$e = \frac{c}{a}$ or $e = \frac{\sqrt{a^2+b^2}}{a}$	$V_1(h+a, k)$ $U_2(h-a, k)$

Axis of Symmetry	End Points of Minor Axis	Foci	Directrices
$y = k$	$B_1(h, k+b)$ $B_2(h, k-b)$	$F_1(h+a\epsilon, k)$ $F_2(h-a\epsilon, k)$	$x = h + \frac{a}{\epsilon}$ $x = h - \frac{a}{\epsilon}$

Asymptotes	End Points of Latera Recta
$\frac{x-h}{a} \pm \frac{y-k}{b} = 0$	$\angle_1 (k + a\epsilon, k \pm b^2/a)$ $\angle_2 (k - a\epsilon, k \pm b^2/a)$

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
73 (cont'd)	The student is not expected to fill out all columns for the general equation form of the hyperbola. However, in any given problem, all entries should be completed for the discussion. SUGGESTED BASIC ASSIGNMENT: MRev... P. 205: Ex. 2acegikmoqsu M...... P. 187: Ex. 2acegikmoqsu		
74	Rotation of the Axes (6-5) Students familiar with matrices may wish to write formulas (6-17) to (6-20) as $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\text{sine}\ \theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$ Refer to appendix in the text. The mnemonic device $\begin{array}{c\|cc} & x' & y' \\ \hline x & \cos\theta & -\sin\theta \\ y & \sin\theta & \cos\theta \end{array}$ is useful in remembering linear equations (6-15, 14, 15, 16) SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 209, 210: Ex. 1acd, 2ace M...... Pp. 191, 192: Ex. 1acd, 2ace	MRev... M......	205-210 187-192

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
75	Simplification of the Equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ Where $B \neq 0$ (6-6) The teacher should develop equation (6-26) very carefully in class. Make sure the students understand the sign convention for this formula thoroughly. Prove in class that choosing the signs of cos 2θ and ctn 2θ identically guarantees that the xy-term can always be eliminated by a rotation through a positive acute angle. The student should not memorize formula (6-26), but should know $\cot 2\theta = \frac{A-C}{B}$ and $\sin\theta = \sqrt{\frac{(1 - \cos 2\theta)}{2}}$ and $\cos\theta = \sqrt{\frac{(1 + \cos 2\theta)}{2}}$. The teacher should develop methods for simplifying the general quadratic equation in two variables for only those cases which have an xy term but no linear term. SUGGESTED BASIC ASSIGNMENT: MRev... P. 215: Ex. 1, also eliminate the xy-term and discuss the equation $xy = 1$ M...... P. 197: Ex. 1, also eliminate xy-term and discuss the equation $xy = 1$	MRev... M.....	210-212 192-194
76	Simplification of the Equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ Where $B \neq 0$ (6-6) (Continued from Lesson 75)	MRev... M.....	212-215 194-197

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
76 (cont'd)	Develop methods for simplifying the general quadratic equation in two variables in which both the xy-term and linear terms appear. SUGGESTED BASIC ASSIGNMENT: MRev... P. 215: Ex. 7 M...... P. 197: Ex. 7		
77	Invariants of the Second Degree Equation (6-7)	MRev... M......	216-218 198-200
78	The Characteristic $4AC - B^2$ and the Discriminant Δ (6-8) SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 221, 222: Ex. 1, 3, 4, 5, 6, 9 M...... P. 202: Ex. 1, 3, 4, 5, 6, 9	MRev... M......	218-220 200-202
79	Review		
80	Test on Chapter 6		

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	PAGE
	UNIT NINE Polar Coordinates		
81	Polar Coordinates of a Point (7-2) Emphasize that the point $P(r, \theta)$ also has the coordinates $P(r, \theta + 2k\pi)$ and $P(-r, \theta + (2k+1)\pi)$, where k is integral. Hence, there does <u>not</u> exist a 1-1 correspondence between the set of points in the plane and the ordered pairs of real numbers thought as polar coordinates. (Refer to top of page 230.) This leads to strange occurrences (see Section 7-12).	MRev... M......	228-230 209-211
	The Relation Between Polar Coordinates and the Cartesian Coordinates (7-3) The teacher should note carefully that the problem of changing the form of an equation from polar coordinates to rectangular coordinates, or conversely. This is equivalent to proving that the set of points defined by polar coordinates is equal to the set of points defined by cartesian coordinates. Since $A = B$ if and only if $A \subseteq B$ <u>and</u> $B \subseteq A$, the student must show <u>both</u> the rectangular form and polar form of an equation <u>can</u> be changed into the other <u>exactly</u>. SUGGESTED BASIC ASSIGNMENT: MRev... P. 230: Ex. 2abef M...... P. 211: Ex. 2abef MRev... P. 234: Ex. 1aeg, 2abcg M...... P. 215: Ex. 1aeg, 2abcg	MRev... M......	230-234 211-215

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82	The Line (7-4) Consider only the cases of the horizontal line, vertical line, and a line through the pole. There is no need to discuss the general line in polar coordinates.	MRev... M......	234-236 215-217
	The Circle (7-5) Discuss the following cases: (1) a circle with center at the pole and radius r (2) a circle with center on the polar axis and passing through the pole (3) a circle with center on the 90° axis and passing through the pole. Do not hold students responsible for the general case of the circle. SUGGESTED BASIC ASSIGNMENT: MRev... P. 236: Ex. 1, 2, 6, 15 M...... P. 217: Ex. 1, 2, 6 MRev... Pp. 238, 239: Ex. 2ac, 4abd, 5abc M...... Pp. 218, 219: Ex. 2ac, 4abd, 5abc	MRev... M......	236-239 217-219
83	Intercept Points (7-6)	MRev... M......	239 219-220
	Symmetry (7-7) Observe that in rectangular coordinates the graph of $y = f(x)$ is symmetrical with respect to the y-axis <u>if and only if</u> $f(-x) = f(x)$. However, in polar coordinates, this condition is not sufficient.	MRev... M......	239-243 220-223

LESSON	SCOPE AND TEACHING SUGGESTIONS	REFERENCE CODE	REFERENCE PAGE
83 (cont'd)	For example, if $f(r,\theta) \neq f(r, -\theta + 2k\pi)$, symmetry with respect to the polar axis may exist. Further tests are necessary. We must check to see whether or not $f(r,\theta) = f(-r, -\theta + [2k+1]\pi)$. If this is the case, then the graph of $f(r,\theta) = 0$ is indeed symmetrical with respect to the polar axis. This is one of the consequences of the fact that there does not exist a 1-1 correspondence between the points in the plane and pairs of polar coordinates. Similar remarks apply to symmetry with respect to the $\frac{\pi}{2}$ -axis and with respect to the pole. SUGGESTED BASIC ASSIGNMENT: MRev... Pp. 247, 248: Discuss the symmetry of the graphs of the equations in exercises ladg(ah) M...... Pp. 227, 228: Discuss the symmetry of the graphs of the equations in exercises ladg(ah)		
84	Extent (7-8)	MRev... M......	243 223-224
	Sketching Curves Representing $p = f(\theta)$ (7-9) SUGGESTED BASIC ASSIGNMENT: MRev... P. 248: Ex. 2ace M...... P. 228: Ex. 2ace	MRev... M......	244 224-227
85	The Conic (7-10) SUGGESTED BASIC ASSIGNMENT: MRev... P. 251: Ex. 1a, 2a M...... P. 231: Ex. 1a, 2a	MRev... M......	249 229-231

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86	Locus Problems (7-11) SUGGESTED BASIC ASSIGNMENT: MRev...P. 253: Ex. 2 M...... P. 233: Ex. 2	MRev... M......	251 232-233
87	Intersection of Curves in Polar Coordinates (7-12) SUGGESTED BASIC ASSIGNMENT: MRev...P. 258: Ex. 1, 3, 5 M,..... P. 239: Ex. 1, 3, 5	MRev... M......	253 233-238
88	Review	MRev... M......	259 239
89	Test		